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MA 425/525
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Homework 7

Due November 15th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. The book is `https://archive.org/details/complex-variables-2ed-dover-1999-fisher/page/n23/mode/2up`.

- Section 3.1 (page 179) #5, 6
- Nonbook problems:
  1. Show that if f is a polynomial with real coefficients, then roots of f are symmetric about the real axis.
  2. For each of $f(z) = z^9 + 5z^2 + 3$ and $f(z) = z^7 + 6z^3 + 7$, find how many zeroes f has in the fourth quadrant and how many in the half plane $\{z\colon \operatorname{Re} z < 0\}$. (This should be quick based on your work above.)
  3. Pick one of the f's above, and find $R > 0$ such that you can use Rouché's theorem (as in Example 4) to determine how many zeroes f has in each of the zones $\{z\colon |z| < R\}$, $\{z\colon R < |z| < R + 1\}$ and $\{z\colon R + 1 < |z|\}$.
  4. Pick one of the f's above and an $R > 0$, make a sketch of the complex plane with the circles $\{z\colon |z| = R\}$ and $\{z\colon |z| = R + 1\}$, and find how many zeroes of f lie in each of the twelve open regions bounded by those circles and the coordinate axes, and how many on each boundary segment between regions.

Hints: For nonbook 1, show and use $\overline{f(z)} = f(\overline{z})$. For nonbook 4, a good choice of f and R makes things easier; it is easiest if all zeroes fall within a single zone. Watch out for zeroes on the axes.